MA114 Summer II 2018
Review Worksheet

Solutions

1. Compute the derivative of each function:

   (a) $f(x) = x^{7.1}$ $\quad f'(x) = 7.1x^{6.1}$

   (b) $g(s) = \tan(s)$ $\quad g'(s) = \sec^2(s)$

   (c) $y = x^2e^{2x}$ $\quad y' = 2xe^{2x} + 2x^2e^{2x} = 2e^{2x}(x^2+x)$

   (d) $z = \cos(7x^2 + 2x + 1)$ $\quad z' = (14x+2)\cos(7x^2+2x+1)$

   (e) $w = \arctan(x)$ $\quad W' = \frac{1}{1+x^2}$

2. Solve the equation for x: (without using the quadratic formula)

$$6x^2 + 13x = 5$$

Rearrange: ~~Set~~ $\quad 6x^2 + 13x - 5 = 0$

Factor LHS: $\quad \cancel{6}(2x+5)(3x-1) = 0$

Set factors to 0: $\quad 2x+5=0 \qquad 3x-1=0$

$x = -5/2 \qquad x = 1/3$

3. Compute these limits:

   (a) $\lim\limits_{x\to\infty} \dfrac{7x^3 + 2x + 1}{5x^3 + 17} \cdot \dfrac{\frac{1}{x^3}}{\frac{1}{x^3}}$ Divide by highest power of x in denominator: x^3

$$= \lim_{x\to\infty} \frac{7 + \frac{2}{x^2} + \frac{1}{x^3}}{5 + \frac{17}{x^3}} = \frac{7+0+0}{5+0} = \boxed{\frac{7}{5}}$$

   (b) $\lim\limits_{x\to\infty} \dfrac{\ln(x)}{x} = \dfrac{\infty}{\infty}$

Apply L'Hôspital's rule:

$$\lim_{x\to\infty} \frac{\ln(x)}{x} = \cancel{\tfrac{d}{dx}} \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = \boxed{0}$$

4. Find each integral:

(a) $\int \frac{x^3+3x+1}{x}\,dx$ $= \int \frac{x^3}{x} + \frac{3x}{x} + \frac{1}{x} dx = \int x^2+3+\frac{1}{x} dx = \boxed{\frac{1}{3}x^3 + 3x + \ln|x| + C}$

(b) $\int (e^x+\cos(x)+\sin(x))\,dx$ $= \boxed{e^x + \sin(x) - \cos(x) + C}$

(c) $\int z^3\cos(z^4+7)\,dz$ Substitute $u = z^4+7$, $du = 4z^3 dz \iff dz = \frac{du}{4z^3}$

$= \int \frac{\cancel{z^3}\cos(u)\,du}{4\cancel{z^3}} = \frac{1}{4}\int \cos(u)du = \boxed{\frac{1}{4}\sin(z^4+7) + C}$

(d) $\int \frac{1}{5-2y}\,dy$ Substitute $u = 5-2y$, $du = -2\,dy \iff dy = -\frac{1}{2}du$

$= -\frac{1}{2}\int \frac{1}{u}du = -\frac{1}{2}\ln|u| + C = \boxed{-\frac{1}{2}\ln|5-2y| + C}$

5. Simplify the given expression:

$$\frac{120\,z^{15}\,(x+2)^{n+1}\,y^{7/2}}{24\,y^3\,(x+2)^n\,z^{27}}$$

Group like terms ; $= \left(\frac{120}{24}\right)\left(\frac{z^{15}}{z^{27}}\right)\left(\frac{y^{7/2}}{y^3}\right)\left(\frac{(x+2)^{n+1}}{(x+2)^n}\right)$

$= \boxed{5\,z^{-12}\,y^{1/2}\,(x+2)}$